लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2022

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Test whether the equation $(\sin 2x - \tan y)\,dx = x\sec^2 y\; dy$ is exact or not?

(b) Find an integrating factor of the differential equation $(2x^2 + y^2 + x)\,dx + xy\,dy = 0$.

(c) Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

(d) Verify if the following pair of functions are independent

$$e^x \quad , \quad 5e^x$$

(e) Given that $y_1(x)$, $y_2(x)$ and $y_3(x)$ are solutions of $\{D^2 + p(x)D + q(x)\}y = 0$, where $D \equiv \dfrac{d}{dx}$. Show that these solutions are linearly independent.

(f) Verify the integrability of the following differential equation:

$$yz\,dx = zx\,dy + y^2dz$$

(g) Determine the order, degree and linearity of the following P.D.E:

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$$

(h) Eliminate the arbitrary functions ϕ and ψ from $z = \phi(x+iy) + \psi(x-iy)$, where $i^2 = -1$.

2. (a) Determine the constant A of the following differential equation such that the equation is exact and solve the resulting exact equation: 4

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

(b) Reduce the equation $\sin y\dfrac{dy}{dx} = \cos x\,(2\cos y - \sin^2 x)$ to a linear equation and hence solve it. 4

3. (a) Using the transformation $u = x^2$ and $v = y^2$ to solve the equation 4

$$xyp^2 - (x^2 + y^2 - 1)\,p + xy = 0 \quad , \quad \text{where } p = \frac{dy}{dx}$$

(b) Solve: $\dfrac{dy}{dx}+\dfrac{ax+hy+g}{hx+by+f}=0$ 4

4. (a) Solve by the method of variation of parameters: 4

$$\frac{d^2y}{dx^2}+a^2y=\cos ax$$

(b) Show that e^x and xe^x are linearly independent solutions of the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0)=1,\ y'(0)=4$. Is it the unique solution? 1+1+1+1

5. (a) Solve: $\{(5+2x)^2D^2-6(5+2x)D+8\}y=8(5+2x)^2$, where $D\equiv\dfrac{d}{dx}$. 4

(b) Solve the following equations: 4

$$\frac{dx}{dt}+4x+3y=t \qquad ; \qquad \frac{dy}{dt}+2x+5y=e^t$$

6. (a) Verify that the following equation is integrable, find its primitive: 5

$$zy\,dx+(x^2y-zx)\,dy+(x^2z-xy)\,dz=0$$

(b) Solve: $(4x^2y-6)\,dx+x^3\,dy=0$ 3

7. (a) Eliminate the arbitrary function ϕ from the relation $z=e^{my}\phi(x-y)$. 3

(b) Solve the PDE by Lagrange's method: 5

$$px(x+y)-qy(x+y)+(x-y)(2x+2y+z)=0$$

8. (a) Find the particular solution of the differential equation 4

$$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y$$

which passes through the curve $xy=4,\ z=0$.

(b) Determine the points $(x,\ y)$ at which the partial differential equation 4

$$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial y\partial x}-\frac{\partial^2 z}{\partial y^2}=0$$

is hyperbolic or parabolic or elliptic.

9. (a) Solve: $(x^2+y^2+z^2)\,dx-2xy\,dy-2xz\,dz=0$ 4

(b) Solve in particular cases: 4

$$\frac{d^2y}{dx^2}+y=\sin 2x \quad ; \quad \text{when } x=0,\ y=0 \text{ and } \frac{dy}{dx}=0$$

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2021

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Test whether the equation $xdx+ydy+\dfrac{xdy-ydx}{x^2+y^2}=0$ is exact or not.

(b) Find an integrating factor of the differential equation $(x\log x)\dfrac{dy}{dx}+y=2\log x$.

(c) Find particular integral of the differential equation $2x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}=\dfrac{1}{x}$.

(d) Find the transformation of the differential equation $x^2\dfrac{d^2y}{dx^2}-5y=\log x$, using the substitution $x=e^z$.

(e) Find complementary function of the differential equation $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}=3x$.

(f) Find the Wronskian of $y_1(x)=e^{-2x}$, $y_2(x)=xe^{-2x}$.

(g) Construct a PDE by eliminating a and b from $z=ae^{-b^2t}\cos bx$.

(h) Determine the order, degree and linearity of the following PDE:

$$\frac{\partial z}{\partial x}=\left(\frac{\partial^2 z}{\partial x^2}\right)^{5/2}+\left(\frac{\partial^2 z}{\partial y^2}\right)^{5/2}$$

(i) Classify the following PDE

$$(1+x^2)\,z_{xx}+(1+y^2)\,z_{yy}+xz_x+yz_y=0$$

into elliptic, parabolic and hyperbolic for different values of x and y.

2. (a) Find an integrating factor of the differential equation 4

$$(2xy^4e^y+2xy^3+y)\,dx+(x^2y^4e^y-x^2y^2-3x)\,dy=0$$

and hence solve it.

(b) Solve: $x\cos x\dfrac{dy}{dx}+y\,(x\sin x+\cos x)=1$ 4

3. (a) Find the curve for which the area of the triangle formed by x-axis, a tangent and the radius vector of the point of tangency is constant and equal to a^2. 4

(b) Using the substitution $u=\dfrac{1}{x}$ and $v=\dfrac{1}{y}$, reduce the equation $y^2(y-px)=x^4p^2$ to Clairaut's form and hence solve it. Here $p\equiv\dfrac{dy}{dx}$. 4

4. (a) Show that each of the functions e^x, e^{4x} and $2e^x-3e^{4x}$ is solution of the differential equation $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=0,\ -\infty<x<\infty$. 2+1+1+1

Are the three independent? If not, find which two of these are independent. Write down a general solution of the equation.

(b) Find the value of h so that the equation $(ax+hy+g)\,dx+(3x+by+f)\,dy=0$ becomes an exact differential equation. 3

5. (a) Solve by the method of variation of parameters: 5

$$(D^2-3D+2)y=e^x(1+e^x)^{-1},\ \text{where } D\equiv\frac{d}{dx}$$

(b) Find particular integral of the differential equation 3

$$(D^2+5D+6)y=e^{-2x}\sin 2x,\quad \text{where } D\equiv\frac{d}{dx}$$

6. (a) Solve in the particular cases: 5

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+5x=0 \text{ giving that } x=1 \text{ and } \frac{dx}{dt}=2 \text{ when } x=0$$

(b) Solve: $\dfrac{d^2y}{dx^2}=x^2\sin x$ 3

7. (a) Solve the following total differential equation: 4

$$yz\,dx+2zx\,dy-3xy\,dz=0$$

(b) Solve: $x^2\dfrac{d^2y}{dx^2}+3x\dfrac{dy}{dx}+y=x\log x$ 4

8. (a) Form a PDE by eliminating the arbitrary function ϕ from 4

$$lx+my+nz=\phi(x^2+y^2+z^2)$$

(b) Solve the partial differential equation by Lagrange's method $x^2p+y^2q=(x+y)z$. 4

9. (a) Find the partial differential equation of planes having equal intercepts along x axis and y axis. 4

   (b) Find $f(y)$ such that the total differential equation $\left(\frac{yz+z}{x}\right)dx - zdy + f(y)\,dz = 0$ is integrable. 4

10. (a) Formulate a PDE from the relation $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. 3

   (b) Find the Wronskian of x and $|x|$ in $[-1, 1]$. 2

   (c) Solve $x^2 \frac{d^2y}{dx^2} - 6y = 0$. 3

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2020

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

DIFFERENTIAL EQUATIONS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Examine whether $\{\cos x\tan y+\cos(x+y)\}dx+\{\sin x\sec^2 y+\cos(x+y)\}dy$ is an exact differential equation.

(b) Show that the functions 1, x and x^2 are linearly independent. Hence find the differential equation whose solutions are 1, x and x^2.

(c) Prove that if f and g are two different solutions of $y'+P(x)y=Q(x)$, then $f-g$ is a solution of the equation $y'+P(x)y=0$.

(d) Show that $\{x(x^2-y^2)\}^{-1}$ is an integrating factor of the differential equation $(x^2+y^2)dx-2xydy=0$.

(e) Find a particular integral of the differential equation

$$(D^2-4D)y=x^2 \text{ where } D\equiv\frac{d}{dx}.$$

(f) Eliminating the arbitrary constants from the following equation form the partial differential equation:

$$z=(a+x)(b+y)$$

(g) Eliminate the arbitrary function f and g from $z=f(x+iy)+g(x-iy)$ where $i^2+1=0$.

(h) Find the order and degree of the following differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3+x^2\left(\frac{dy}{dx}\right)^4=4$$

2. (a) Obtain the general solution of the differential equation 4

$$xdy-ydx+a(x^2+y^2)dx=0$$

(b) Determine the constant A so that the following differential equation is exact and hence solve the resulting equation: 4

$$\left(\frac{Ay}{x^3}+\frac{y}{x^2}\right)dx+\left(\frac{1}{x^2}-\frac{1}{x}\right)dy=0$$

3. (a) Given that $y=x+1$ is a solution of $[(x+1)^2D-3(x+1)D+3]y=0$, find a linearly independent solution by reducing the order. Hence determine the general solution. $\left(D\equiv\frac{d}{dx}\right)$ 5

(b) Find an integrating factor of the following differential equation 3

$$x\frac{dy}{dx}+\sin 2y=x^4\cos^2 y$$

4. (a) Obtain complete primitive and singular solution of 3

$$y=px+(1+p^2)^{1/2}$$

(b) Solve: $p^2+px=xy+y^2$ 5

5. (a) Show that e^x and xe^x are linearly independent solutions of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0)=1$, $y'(0)=4$. Is it unique solution? Over which interval is it defined? 1+1+1+1+1

(b) The complementary function of $\frac{d^2y}{dx^2}+y=\cos x$ is $A\sin x+B\cos x$, where A and B are constants. Find a particular integral. 3

6. (a) Apply the method of variation of parameters to solve the following equation: 6

$$x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^2\log x$$

(b) Fill in the blank: 2

In the 'method of variation of parameter' if $y=A\ f_1(x)+B\ f_2(x)$ be the complementary function then the complete primitive is $y=\phi(x)f_1(x)+\psi(x)f_2(x)$ provided …………..

7. (a) Solve: $\frac{dx}{dt}=-2x+7y$, $\frac{dy}{dt}=3x+2y$ subject to the conditions $x(0)=9$ and $y(0)=-1$. 4

(b) Solve: $\frac{d^2y}{dx^2}+y=\sin 2x$ given that $y=0$ and $\frac{dy}{dx}=0$ when $x=0$. 4

8. (a) Verify that the following equation is integrable and find its primitive: 5

$$zydx+(x^2y-zx)dy+(x^2z-xy)dz=0.$$

(b) Find a complete integral of the following partial differential equation by Charpit's method: $z=p+q$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. 3

9. (a) Find the particular solution of the differential equation 5

$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y$ which passes through the curve $xy=4$, $z=0$.

(b) Classify the partial differential equation 3

$$\frac{\partial^2 z}{\partial x^2}+(1-x)\frac{\partial^2 z}{\partial y^2}=0$$

into elliptic, parabolic and hyperbolic for different values of *x*.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2019

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Find an integrating factor of the differential equation $y^2+(x-\frac{1}{xy})\frac{dy}{dx}=0$. 2

(b) What is the Clairaut's form for first order ordinary differential equation? Write down the general solution of it. 2

(c) Find the Wronskian of x and $-x$ in $(-1, 1)$. 2

(d) Find $\frac{1}{(D-1)^2}(x^2e^{3x})$, where $D=\frac{d}{dx}$. 2

(e) Eliminate the arbitrary function f and F from the relation $y=f(x-at)+F(x+at)$. 2

(f) Determine the order, degree and linearity of the following PDE: 2

$$xy\left(\frac{\partial^2 z}{\partial x^2}\right)^2-2\frac{\partial z}{\partial y}=1$$

(g) Classify the following partial differential equation: 2

$$z_{xx}-2\sin x\, z_{xy}-\cos^2 x\, z_{yy}-\cos x\, z_y=0$$

(h) Verify the condition of integrability for the equation 2

$$(2x+y^2+2xz)\,dx+2xy\,dy+x^2\,dz=0$$

2. (a) Examine whether the following differential equation is exact and if so find the general solution. 4

$$(\cos y+y\cos x)\,dx+(\sin x-x\sin y)\,dy=0$$

(b) Obtain the general solution of the differential equation 4

$$\frac{d^2y}{dx^2}+4y=\sin^2 x$$

3. (a) Solve the following differential equation 5

$$(px-y)(x-py)=2p \text{ where } p=\frac{dy}{dx}.$$

(b) Prove that x, x^2 and x^4 are independent solution of the differential equation 3

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0$$

Write down the general solution also.

4. (a) Solve: $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2e^{3x}$ 4

(b) Solve: $(x^2D^2-xD+4)y=\cos(\log x)+x\sin(\log x)$, where $D=\frac{d}{dx}$. 4

5. (a) Solve: $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0$; $\frac{dy}{dt}+5x+3y=0$ 4

(b) Solve $(D^2+2D+1)y=e^{-x}\log x$ by the method of variation of parameters. 4

6. (a) Solve: $\frac{a^4dx}{(b-c)yz}=\frac{b^3dy}{(c-a)zx}=\frac{c^2dz}{(a-b)xy}$ 5

(b) Find particular integral of the differential equation $(D^2+49)y=x\sin x$, where $D\equiv\frac{d}{dx}$. 3

7. (a) Eliminate a, b from the relation: 3

7.

$$z=ax^2+by^2+ab$$

(b) Solve the partial differential equation by Lagrange's method: 5

$$y^2(x-y)p+x^2(y-x)q=z(x^2+y^2)$$

8. (a) Find a complete integral of the following partial differential equation by Charpit's method: 5

$$pxy+pq+qy=yz$$

(b) Form a partial differential equation by eliminating the arbitrary function from the relation: $x+y+z=f(x^2+y^2+z^2)$ 3

9. (a) Solve: $(x^2+y^2+z^2)dx-2xy\,dy-2xz\,dz=0$ 4

(b) Solve: $(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)$, given that $y=x$ is a solution of its reduced equation. 4

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